

The Gluon Propagator in the Coulomb Gauge

A. Andraši

'Rudjer Bošković' Institute, Zagreb, Croatia

Abstract

We give the results for all the one-loop propagators, including finite parts, in the Coulomb gauge. In finite parts we find new non-rational functions in addition to the single logarithms of the Feynman gauge. Of course, the two gauges must agree for any gauge invariant function.

PACS: 11.15.Bt; 11.10.Gh

Keywords: Coulomb gauge; Gluon propagator

Electronic address: aandrasi@rudjer.irb.hr

1. Introduction

The non-covariant axial and Coulomb gauges have more direct physical interpretation than the covariant gauges because their propagators are closely related to the polarization states of real spin-1 particles. The relevant diagrams in the Coulomb gauge are not plagued by ghosts. Also the time-time component of the gluon propagator provides a long-range confining force [1], [2]. The Hamiltonian for non-Abelian gauge theory in the Coulomb gauge has been known for some time in its continuum version [3]. The Coulomb gauge in the Hamiltonian formalism is manifestly unitary. The main point in its favour is that problems concerned with the definition of the axial gauge integrals like

$$\int d^4k \frac{1}{(n \cdot k)^2} \quad (1)$$

do not appear in the definition of integrals like

$$\int d^4k \frac{1}{K^2} \dots \quad (2)$$

in the Coulomb gauge. However, there are disadvantages. The naive Coulomb gauge Feynman rules in non-Abelian gauge theory give rise to ambiguous integrals, in addition to the usual ultra-violet divergences [4]. At one loop order and above there are integrals like

$$\int \frac{d^3P}{(2\pi)^3} \int \frac{dp_0}{(2\pi)} \frac{p_0}{p_0^2 - P^2 + i\eta} \times \frac{1}{(P - K)^2}. \quad (3)$$

There is no regularization procedure for the energy divergence in p_0 within the standard dimensional regularization scheme. This integral and similar more complicated divergences in higher order diagrams have been the subject of the study [5], [6], where systematic cancellations have been found. However, no general proof exists that controls all divergences [7]. Formally such integrals are assigned value zero. The Coulomb gauge has been extensively studied in the phase space formalism by D. Zwanziger [8] in the Euclidean space. The ultra-violet divergent parts of the proper two-point functions have been calculated and found to observe the Ward identities. In addition, a more powerful Ward identity holds in the Coulomb gauge than is available in covariant gauges. In this paper we give the results for the complete propagator to order g^2 including finite parts in Minkowski space.

2. The Coulomb gauge in the phase-space formalism

We use the phase-space formalism in order to avoid the ambiguous integrals like (3). Let the generating functional of the Green's functions be

$$Z(j, J) = \int d[f] d[A] [J^\mu A_\mu + j^{\mu\nu} f_{\mu\nu}] \exp[-i \int d^4x L], \quad (4)$$

where J, j are sources, L the Lagrangian density and f and A are the fields [9].

$$L = -\frac{1}{4} f_{\mu\nu}^a f^{a\mu\nu} + \frac{1}{2} f^{a\mu\nu} F_{\mu\nu}^a, \quad (5)$$

and

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c. \quad (6)$$

The Greek indices run from 0 to 3 and Latin indices denote spatial dimensions ($i = 1, 2, 3$), a, b, c , are colour indices. Instead of setting the source j to zero as in Lagrangian formalism, we keep some components of j . We write out L as

$$L = -\frac{1}{4} (f_{ij}^a)^2 + \frac{1}{2} (f_{0i}^a)^2 + \frac{1}{2} f_{ij}^a F_{ij}^a - f_{0i}^a F_{0i}^a \quad (7)$$

and set $j_{ij} = 0$ in order to perform the Feynman integral over f_{ij} . We denote $f_{0i} = E_i$. The Lagrangian becomes

$$L = -\frac{1}{4} (F_{ij}^a)^2 + \frac{1}{2} (E_i^a)^2 - E_i^a F_{0i}^a \quad (8)$$

where

$$E_i^a F_{0i}^a = E_i^a [\partial_0 A_i^a - \partial_i A_0^a - g f^{abc} A_0^b A_i^c]. \quad (9)$$

The field E_i is the momentum conjugate to A_i . We have L expressed in terms of momenta and linear terms in derivatives. It is first order in time derivatives. Adding the gauge fixing term

$$\frac{1}{2\alpha} (\partial_i A_i)^2 \quad (10)$$

we can deduce the propagators from the quadratic part of the Lagrangian. The propagator matrix is the inverse of the matrix of these quadratic parts, and is given by the following 7×7 matrix, whose rows and columns are labeled by $A_1, A_2, A_3 ; A_0 ; E_1, E_2, E_3$:

	A_j	A_0	E_n
A_i	$-T_{ij}/k^2 + \alpha L_{ij}/K^2$	$\alpha k_0 K_i/(K^2)^2$	$-ik_0 T_{in}/k^2$
A_0	$\alpha k_0 K_j/(K^2)^2$	$1/K^2 + \alpha k_0^2/(K^2)^2$	iK_n/K^2
E_m	$-ik_0 T_{mj}/k^2$	iK_m/K^2	$-T_{mn}K^2/k^2$

where

$$\begin{aligned}
T_{ij} &\equiv \delta_{ij} - L_{ij}, & L_{ij} &\equiv K_i K_j / K^2, \\
k^2 &= k_0^2 - K^2.
\end{aligned} \tag{11}$$

The Coulomb gauge propagators are obtained by setting $\alpha = 0$.

3. The proper two-point functions

The method of evaluation of Coulomb gauge integrals is explained in Appendix A. Here we list the results. The constants used throughout the paper are

$$\epsilon = 4 - d \tag{12}$$

where d is the dimension of space-time and the coupling parameter is

$$c\delta_{ab} = \frac{ig^2}{16\pi^2} C_G \delta_{ab} \tag{13}$$

.

The transverse gluon two-point function

There are two non-vanishing graphs contributing to the transverse gluon propagator. The graph shown in Fig.1 gives

$$\begin{aligned}
\Gamma_1^{A_i A_j} &= c\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}} \times 2^{-\epsilon} \left(1 + \frac{77}{30}\epsilon\right) \\
&\times \left\{ \frac{12}{15} K^2 \delta_{ij} - \frac{16}{15} K_i K_j + \frac{4}{15} \epsilon K_i K_j \right\}
\end{aligned} \tag{14}$$

The graph in Fig.2 contributes

$$\Gamma_2^{A_i A_j} = c(M K_i K_j + N K^2 \delta_{ij}) \tag{15}$$

where

$$\begin{aligned}
M = & \frac{31}{15}\Gamma(\frac{\epsilon}{2}) - \frac{4}{3}\ln(\frac{-k^2 - i\eta}{\mu^2}) + \frac{3}{10}\ln\frac{K^2}{\mu^2} \\
& + \frac{1}{4}[-\frac{K^4}{k_0^2} + 18\frac{k_0^2 k^2}{K^2} + 9\frac{k_0^2 k^4}{K^4} + k_0^2] \times D \\
& + \frac{1}{4}[-\frac{K^3}{k_0^3} + 18\frac{k_0 k^2}{K^3} + 9\frac{k_0 k^4}{K^5} + \frac{k_0}{K}] \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \times \ln \frac{K^2}{(-k^2 - i\eta)} \\
& - \frac{1}{2K^2}[3k_0^2 - 5K^2 + 9\frac{k_0^4}{K^2} - \frac{K^4}{k_0^2}] \ln \frac{K^2}{(-k^2 - i\eta)} \\
& - \frac{\ln 2}{K^2}[6\frac{k_0^4}{K^2} + 9k^2 - \frac{1}{5}K^2 + 3\frac{k^4}{K^2} - \frac{K^4}{k_0^2}] \\
& + 22\frac{k_0^2}{K^2} - 16 - \frac{1}{9} - \frac{8}{15} + 4 \times \frac{77}{225}
\end{aligned} \tag{16}$$

$$\begin{aligned}
N = & -\frac{1}{3K^2}(k_0^2 + \frac{27}{5}K^2)\Gamma(\frac{\epsilon}{2}) + \frac{1}{3K^2}(k_0^2 + 8K^2)\ln(\frac{-k^2 - i\eta}{\mu^2}) - \frac{13}{15}\ln\frac{K^2}{\mu^2} \\
& + \frac{1}{4}[\frac{K^4}{k_0^2} - \frac{k_0^2 k^2}{K^2}(14 + \frac{K^2}{k^2} + 3\frac{k^2}{K^2})] \times D \\
& + \frac{1}{4}[\frac{K^3}{k_0^3} - \frac{k_0 k^2}{K^3}(14 + \frac{K^2}{k^2} + 3\frac{k^2}{K^2})] \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \times \ln \frac{K^2}{(-k^2 - i\eta)} \\
& + \frac{1}{2K^2}(9k^2 + 6k_0^2 + K^2 + 3\frac{k^4}{K^2} - \frac{K^4}{k_0^2}) \ln \frac{K^2}{(-k^2 - i\eta)} \\
& + \frac{\ln 2}{K^2}(9k^2 + 6k_0^2 + 3\frac{k^4}{K^2} - \frac{K^4}{k_0^2} - \frac{11}{15}K^2) \\
& - 16\frac{k_0^2}{K^2} + 10 + \frac{2}{9}
\end{aligned} \tag{17}$$

The non-rational structure D which appears in the results for the proper two-point functions is in the integral form

$$D = \int_0^1 dx \frac{x^{-\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} \ln(1 - x). \tag{18}$$

In the region $k_0 > K$

$$D = \frac{1}{k_0 K} \left\{ Li_2\left(\frac{k_0 - K + i\eta}{k_0 + K - i\eta}\right) - Li_2\left(\frac{k_0 + K - i\eta}{k_0 - K + i\eta}\right) \right. \\ \left. + \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \times \ln \frac{k^2 + i\eta}{K^2} - i\pi \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \right\}. \quad (19)$$

In the region $K > k_0$

$$D = \frac{1}{k_0 K} \left\{ Li_2\left(\frac{K - k_0 - i\eta}{K + k_0 - i\eta}\right) - Li_2\left(\frac{K + k_0 - i\eta}{K - k_0 - i\eta}\right) \right. \\ \left. + \ln \frac{K + k_0 - i\eta}{K - k_0 - i\eta} \times \ln\left(\frac{-k^2 - i\eta}{k_0^2}\right) + i\pi \ln \frac{K + k_0 - i\eta}{K - k_0 - i\eta} \right\} \\ - \frac{2}{k_0 K} \left[Li_2\left(-\frac{k_0}{K - i\eta}\right) - Li_2\left(\frac{k_0}{K - i\eta}\right) \right] + \frac{i\pi}{k_0 K} \ln \frac{K^2}{(-k^2 - i\eta)} \quad (20)$$

where

$$Li_2(x) = - \int_0^x \frac{\ln(1-z)}{z} dz \quad (21)$$

is the Spence function and k_0 and K are the lengths of the respective vectors. The two expressions for D in (19) and (20) are connected as analytic continuations of each other with the relation (B1).

The $A_i A_0$ transition

The whole contribution to the $A_i A_0$ transition to order g^2 comes from the graph in Fig.3a.

$$\Gamma^{A_i A_0} = ck_0 K_i \times Z \quad (22)$$

$$Z = -\frac{1}{3} \Gamma\left(\frac{\epsilon}{2}\right) \left(\frac{-k^2 - i\eta}{\mu^2}\right)^{-\frac{\epsilon}{2}} \\ + \frac{k^2}{2K^2} (2k_0^2 + k^2) \times D \\ + \frac{k^2}{2k_0 K^3} (2k_0^2 + k^2) \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \times \ln \frac{K^2}{(-k^2 - i\eta)} \\ - 3 \frac{k^2}{K^2} \ln \frac{K^2}{(-k^2 - i\eta)} - 6 \frac{k^2}{K^2} \ln 2 + 6 \frac{k_0^2}{K^2} - \frac{53}{9} \quad (23)$$

The graph shown in Fig.3b contains integrals like the one in (3). Formally such integrals are assigned value zero.

The time-time component of the two-point function

Two graphs contribute to the $A_0 A_0$ function. The graph shown in Fig.4a gives

$$\begin{aligned}
\Gamma_a^{A_0 A_0} = & c \left\{ \Gamma\left(\frac{\epsilon}{2}\right) \left(\frac{-k^2 - i\eta}{\mu^2}\right)^{-\frac{\epsilon}{2}} \times \left(\frac{1}{2}k_0^2 + \frac{5}{6}K^2 + \frac{\epsilon}{12}k^2 + \frac{\epsilon}{6}k_0^2 + \frac{17}{18}\epsilon K^2\right) \right. \\
& - 2^{-\epsilon} \left(\frac{5}{3} + \frac{28}{9}\epsilon\right) \Gamma\left(\frac{\epsilon}{2}\right) K^2 \left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}} \\
& + \frac{1}{2}k^4 \times D \\
& + \frac{k^4}{2k_0 K} \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \times \ln \frac{K^2}{(-k^2 - i\eta)} \\
& \left. - k_0^2 \ln \frac{K^2}{(-k^2 - i\eta)} - 2(\ln 2 - 1)k_0^2 \right\} \quad (24)
\end{aligned}$$

The graph in Fig.4b contributes

$$\begin{aligned}
\Gamma_b^{A_0 A_0} = & c \left\{ \Gamma\left(\frac{\epsilon}{2}\right) \left(\frac{-k^2 - i\eta}{\mu^2}\right)^{-\frac{\epsilon}{2}} \times \left(\frac{1}{3}K^2 - \frac{1}{2}k^2 - \frac{\epsilon}{4}k^2 + \frac{11}{18}\epsilon K^2\right) \right. \\
& - \frac{1}{3}\Gamma\left(\frac{\epsilon}{2}\right) K^2 \left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}} - K^2 \left(\frac{10}{9} - \frac{2\ln 2}{3}\right) \\
& + k^2 k_0^2 \times D \\
& + \frac{k_0 k^2}{K} \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \times \ln \frac{K^2}{(-k^2 - i\eta)} \\
& \left. - (2k^2 + K^2) \ln \frac{K^2}{(-k^2 - i\eta)} - 2(2k^2 + K^2)(\ln 2 - 1) \right\} \quad (25)
\end{aligned}$$

We can verify that the complete proper two-point functions satisfy the 't Hooft identity [10]

$$k_0^2 \Gamma^{A_0 A_0} - 2k_0 K_i \Gamma^{A_i A_0} + K_i K_j \Gamma^{A_i A_j} = 0 \quad (26)$$

and the stronger Zwanziger identity [8]

$$k_0 \Gamma^{A_0 A_0} = K_i \Gamma^{A_i A_0}. \quad (27)$$

The remaining graphs contain the conjugate field E_i as the external leg.

$E_i A_j$ graph

The graph in Fig.5 vanishes as the energy divergence.

$E_i A_0$ graph

The graph in Fig.6 gives

$$\Gamma^{E_i A_0} = c \left(\frac{K^2}{\mu^2} \right)^{-\frac{\epsilon}{2}} \left\{ \frac{4}{3} - 2^{-\epsilon} \Gamma\left(\frac{\epsilon}{2}\right) \left(\frac{2}{3} + \frac{13\epsilon}{9} \right) \right\} \times (2iK_i) \quad (28)$$

$E_i E_j$ graph

The graph in Fig.7 contributes

$$\Gamma^{E_i E_j} = -2c \left(\frac{K^2}{\mu^2} \right)^{-\frac{\epsilon}{2}} \times \left\{ 2^{-\epsilon} \left(\frac{2}{3} + \frac{13\epsilon}{9} \right) \Gamma\left(\frac{\epsilon}{2}\right) \delta_{ij} - \frac{4}{3} \frac{K_i K_j}{K^2} \right\} \quad (29)$$

4. The gluon propagator to order g^2

We form the 7×7 matrix of free and order g^2 proper two-point functions. The inverse of this matrix gives the gluon propagator to order g^2 .

The $A_0 A_0$ propagator

The time-time component of the gluon propagator to order g^2 is [11]

$$D^{A_0 A_0} = \frac{1}{K^4} [\Gamma^{A_0 A_0} + iK_n \Gamma^{A_0 E_n}] + \frac{iK_m}{K^4} [\Gamma^{E_m A_0} + iK_n \Gamma^{E_m E_n}] \quad (30)$$

or explicitly

$$\begin{aligned} D^{A_0 A_0} = c(K^2)^{-2} \times & \left\{ \frac{11}{3} \Gamma\left(\frac{\epsilon}{2}\right) K^2 - \frac{5}{3} K^2 \ln \frac{(-k^2 - i\eta)}{\mu^2} - 2K^2 \ln \frac{K^2}{\mu^2} \right. \\ & + \frac{1}{2} k^2 (k^2 + 2k_0^2) \times D \\ & + \frac{k^2}{2k_0 K} (k^2 + 2k_0^2) \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \times \ln \frac{K^2}{(-k^2 - i\eta)} \\ & \left. - (3k_0^2 - K^2) \ln \frac{K^2}{(-k^2 - i\eta)} - (6k_0^2 + 2K^2) \ln 2 + 6k_0^2 + \frac{31}{9} K^2 \right\} \quad (31) \end{aligned}$$

The ultraviolet divergent part of (31) gives the gauge invariant Coulomb field renormalization factor [12].

The $A_i A_j$ propagator

The transverse gluon propagator to order g^2 is

$$D^{A_i A_j} = \frac{1}{k^4} T_{am} \Gamma^{A_m A_n} T_{nj} - \frac{k_0^2}{k^4} T_{am} \Gamma^{E_m E_n} T_{nj} \quad (32)$$

or explicitly

$$\begin{aligned}
D^{A_i A_j} = & \frac{c}{k^2 + i\eta} (\delta_{ij} - \frac{K_i K_j}{K^2}) \times \{ \Gamma(\frac{\epsilon}{2}) - \frac{4}{3} \ln \frac{K^2}{\mu^2} + \frac{1}{3} \ln(\frac{-k^2 - i\eta}{\mu^2}) \\
& - \frac{K^2}{4} [\frac{K^2 + k_0^2}{k_0^2} + \frac{k_0^2}{K^2} (14 + 3 \frac{k^2}{K^2})] \times D \\
& - \frac{K}{4k_0} [\frac{K^2 + k_0^2}{k_0^2} + \frac{k_0^2}{K^2} (14 + 3 \frac{k^2}{K^2})] \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \times \ln \frac{K^2}{(-k^2 - i\eta)} \\
& + \frac{1}{2} (15 + 3 \frac{k^2}{K^2} + \frac{K^2}{k_0^2}) \ln \frac{K^2}{(-k^2 - i\eta)} + (3 \frac{k^2}{K^2} + \frac{K^2}{k_0^2} + \frac{37}{3}) \ln 2 - \frac{92}{9} \} \quad (33)
\end{aligned}$$

5. The Slavnov-Taylor identity

Although ghosts are absent from the S-matrix elements they are necessary to formulate the Slavnov-Taylor identities [13],[14]. Diagrammatically they are shown for the self-energy in Fig.8. Algebraically they are

$$k_0 \Gamma^{A_0 A_j} - K_i \Gamma^{A_i A_j} = (K^2 \delta_{ij} - K_i K_j) \Gamma^{C A_i} \quad (34)$$

The diagrams involving ghost-source vertices on the right-hand side are shown in Fig.9a and Fig.9b. The diagram in Fig.9a vanishes as the energy divergence in p_0 . The diagram in Fig.9b contributes

$$\Gamma^{C A_i} = -2c (\frac{K^2}{\mu^2})^{-\frac{\epsilon}{2}} K_i \{ -\frac{4}{3} + 2^{-\epsilon} \Gamma(\frac{\epsilon}{2}) (\frac{2}{3} + \frac{13\epsilon}{9}) \} \quad (35)$$

so the identity is satisfied trivially as implied by (26) and (27).

6. Discussion

We have checked the consistency of the Coulomb gauge to order g^2 including finite parts. The time-time component of the gluon propagator in the Coulomb gauge is believed to provide a long-range confining force. There are two interesting limits of eq.(31). In the Zwanziger picture [8] $g^2 D_{00}$ gives the instantaneous part $V_Z(R)$, which is called the color-Coulomb potential. (Here D_{00} is the time-time component of the gluon propagator). The instantaneous color-Coulomb potential $V_Z(R)$ at large R may serve as an order parameter.

$$K_{Coul} \equiv \lim_{R \rightarrow \infty} \frac{V_Z(R)}{R} \quad (36)$$

A non-zero value of K_{Coul} would be the signal for color confinement. The potential is separated out in momentum space by

$$V_Z(K) = \lim_{k_0 \rightarrow \infty} g^2 D^{A_0 A_0}(k_0, K) \quad (37)$$

where we have written $V_Z(K)$ for the Fourier transform of $V_Z(R)$. The limit $k_0 \rightarrow \infty$ of eq.(31) is

$$\lim_{k_0 \rightarrow \infty} D^{A_0 A_0}(k_0, K) = \frac{c}{K^2} \left\{ \frac{11}{3} \Gamma\left(\frac{\epsilon}{2}\right) - \frac{11}{3} \ln \frac{K^2}{\mu^2} - i\pi - \frac{28}{3} \ln 2 + \frac{103}{9} - 2 \ln \frac{K}{k_0} \right\} \quad (38)$$

and it is not independent of k_0 . There appears to be a difference in the dominant term in (38) and equation (37) of A. Cucchieri and D. Zwanziger [15]. This difference arises because of the statement near the end of Appendix B in [15] that I_2 is finite, and "as a result" I_2 vanishes in the limit $k_0 \rightarrow \infty$. However, the finiteness of I_2 does not imply anything about the behaviour as $k_0 \rightarrow \infty$. In fact, on calculating I_2 , we find that the dominant term as $k_0 \rightarrow \infty$ is $-4/3 \ln(k_0^2/K^2)$. With this value, there is no contradiction with (38) in this paper.

Although the limit as $k_0 \rightarrow \infty$ is not finite, A. Cucchieri and D. Zwanziger [15] have argued that an unambiguous instantaneous part may be defined by using renormalization group arguments.

The limit $k_0 \rightarrow 0$ is naturally related to the definition of the quark-antiquark potential. It follows from considering a rectangular Wilson loop with sides of length T in the time direction (where $T \rightarrow \infty$) and L in the space direction. In the Coulomb gauge the main contribution comes from the D_{00} component of the propagator (where $k_0 \rightarrow 0$) attached to the two time-like sides. The $k_0 \rightarrow 0$ limit of eq.(31) is

$$\lim_{k_0 \rightarrow 0} D^{A_0 A_0}(k_0, K) = \frac{c}{K^2} \left\{ \frac{11}{3} \Gamma\left(\frac{\epsilon}{2}\right) - \frac{11}{3} \ln \frac{K^2}{\mu^2} + \frac{31}{9} \right\} \quad (39)$$

leading to the quark-antiquark potential

$$V(R) = -2\pi^2 g_r^2(\mu) \frac{1}{R} \left\{ 1 + \frac{g^2 C_G}{16\pi^2} \left[\frac{31}{9} + \frac{11}{3} \gamma + \frac{11}{3} \ln(\mu R)^2 \right] \right\} \quad (40)$$

where γ is the Euler's constant, $g_r(\mu)$ is the running coupling constant. If we assume the relation

$$R \times \mu = 1 \quad (41)$$

$g_r(\mu)$ becomes R dependent. We suppose that the exact $g_r(\frac{1}{R})$ tends to zero as $R \rightarrow 0$ and $g_r(\frac{1}{R}) \rightarrow \infty$ for $R \rightarrow \infty$.

Acknowledgements

It is a privilege to thank Prof. J. C. Taylor who gave me invaluable advice and encouragement which made this work possible. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 0098003.

Appendix A

We use two basic integrals for evaluation of the Coulomb gauge integrals.

$$A = \int d^{4-\epsilon} p \frac{1}{p^2 + i\eta} \cdot \frac{1}{(k-p)^2 + i\eta}$$

$$= \frac{1}{2} i\pi \int_0^1 dy \int d^{3-\epsilon} P \{P^2 - 2P \cdot Ky - yk^2 + y^2 k_0^2 - i\eta\}^{-\frac{3}{2}} \quad (A1)$$

$$B = \int d^{4-\epsilon} p \frac{p_0}{(p^2 + i\eta)[(k-p)^2 + i\eta]}$$

$$= \frac{1}{2} i\pi k_0 \int_0^1 y dy \int d^{3-\epsilon} P \{P^2 - 2P \cdot Ky - yk^2 + y^2 k_0^2 - i\eta\}^{-\frac{3}{2}} \quad (A2)$$

As an example we evaluate the integral

$$X_{ij} = \int d^{4-\epsilon} p \frac{p_0}{p^2 + i\eta} \cdot \frac{1}{(k-p)^2 + i\eta} \cdot \frac{P_i P_j}{P^2} \quad (A3)$$

Applying (A2)

$$X_{ij} = \frac{1}{2} i\pi k_0 \int_0^1 y dy \int d^{3-\epsilon} P \frac{P_i P_j}{P^2} \cdot \frac{1}{[P^2 - 2P \cdot Ky - yk^2 + y^2 k_0^2 - i\eta]^{\frac{3}{2}}} \quad (A4)$$

Combining the denominators with the Feynman parameter x

$$X_{ij} = ik_0 \pi^{\frac{1}{2}} \Gamma(\frac{5}{2}) \int_0^1 dx x^{\frac{1}{2}} \int_0^1 y dy \int d^{3-\epsilon} P \frac{P_i P_j}{[P^2 - 2P \cdot Kxy - xyk^2 + y^2 x k_0^2 - i\eta x]^{\frac{5}{2}}} \quad (A5)$$

Now it is easy to perform the $d^{3-\epsilon} P$ and integration over the parameter y giving X_{ij} in the integral form.

$$C^{-1} X_{ij} = \frac{1}{6} \Gamma(\frac{\epsilon}{2}) (K^2)^{-\frac{\epsilon}{2}} \delta_{ij} - \frac{1}{3} \frac{K_i K_j}{K^2} + \frac{1}{3} \delta_{ij} (\frac{13}{6} - \ln 2)$$

$$+ (\frac{1}{4} \delta_{ij} - \frac{K_i K_j}{K^2}) k^2 \{ \int_0^1 dx \frac{x^{\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} + k^2 \int_0^1 dx \frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^2} \ln \frac{K^2(1-x)}{(-k^2 - i\eta)} \}$$

$$+ \frac{K_i K_j}{K^2} k_0^2 \{ \frac{1}{2} \int_0^1 dx \frac{x^{\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} + k^2 \int_0^1 dx \frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^2} \}$$

$$+k^4 \int_0^1 dx \frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^3} \ln \frac{(1-x)K^2}{(-k^2 - i\eta)} \}$$

where

$$C = ik_0 \pi^{\frac{4-\epsilon}{2}}. \quad (A6)$$

The integrals in (A6) are

$$\int_0^1 dx \frac{x^{\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} = -\frac{2}{K^2} + \frac{k_0}{K^3} \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \quad (A7)$$

$$\int_0^1 dx \frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^2} = \frac{1}{k^2 K^2} - \frac{1}{2k_0 K^3} \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \quad (A8)$$

$$\int_0^1 dx \frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^3} = \frac{1}{2K^2 k^4} - \frac{1}{4k_0^2 K^2 k^2} - \frac{1}{8k_0^3 K^3} \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \quad (A9)$$

$$\begin{aligned} T &= \int_0^1 dx \frac{x^{\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} + k^2 \int_0^1 dx \frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^2} \ln \frac{K^2(1-x)}{(-k^2 - i\eta)} \\ &= \frac{2}{K^2} (\ln 2 - 1) + \left[\frac{1}{K^2} - \frac{k^2}{2k_0 K^3} \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} \right] \times \ln \frac{K^2}{(-k^2 - i\eta)} - \frac{k^2}{2K^2} D \end{aligned} \quad (A10)$$

$$\begin{aligned} E &= \int_0^1 dx \frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^3} \ln(1-x) \\ &= \frac{k_0^2 + K^2}{2k_0^2 K^2 k^4} \ln 2 - \frac{1}{2K^2 k^4} - \frac{1}{2K k_0 k^4} \ln \frac{k_0 + K - i\eta}{k_0 - K + i\eta} - \frac{1}{8K^2 k_0^2} D \end{aligned} \quad (A11)$$

where D was defined in (18) and the explicit result given in (19) and (20).

Appendix B

The expressions for D in (19) for $k_0 > K$ and in (20) for $K > k_0$ ought to be connected by analytic continuation. It is easy to see this happens using the following relation between the Spence functions.

$$\begin{aligned} &Li_2\left(\frac{x-1+i\eta}{x+1-i\eta}\right) - Li_2\left(\frac{x+1-i\eta}{x-1+i\eta}\right) + Li_2\left(\frac{1+x-i\eta}{1-x-i\eta}\right) - Li_2\left(\frac{1-x-i\eta}{1+x-i\eta}\right) \\ &+ 2Li_2(-x-i\eta) - 2Li_2(x+i\eta) + \ln \frac{x+1-i\eta}{x-1+i\eta} \times \ln(x^2) \\ &- i\pi \ln \frac{x+1-i\eta}{x-1+i\eta} + i\pi \ln(x^2) + \pi^2 = 0 \end{aligned} \quad (B1)$$

where

$$Li_2(x) = - \int_0^x \frac{\ln(1-z)}{z} dz \quad (B2)$$

References

- [1] N. Gribov, Nucl. Phys. **B139** (1978) 1;
- [2] D. Zwanziger, Nucl. Phys. **B485** (1997) 185;
- [3] N. Christ and T. D. Lee, Phys. Rev. **D22** (1980) 939;
- [4] H. Cheng and E. C. Tsai, Phys. Lett. **B176** (1986) 130;
- [5] P. Doust and J. C. Taylor, Phys. Lett. **197** (1987) 232;
- [6] P. Doust, Ann. of Phys. **177** (1987) 169;
- [7] J. C. Taylor, in Physical and Nonstandard Gauges, Proceedings, Vienna, Austria 1989, P.Gaigg, W. Kummer and M. Schweda (Eds.);
- [8] D. Zwanziger, Nucl. Phys. **B518** (1998) 237;
- [9] J. C. Taylor, private communication;
- [10] G. 't Hooft, Nucl. Phys. **B35** (1971) 167;
- [11] A. Andraši, Europhys. Lett. **Vol.66**, (3), (2004) 338;
- [12] J. Frenkel and J. C. Taylor, Nucl. Phys. **B109** (1976) 439;
- [13] J. C. Taylor, Nucl. Phys. **B33** (1971) 436;
- [14] A. Slavnov, Theor. and Math. Phys. **10** (1972) 99;
- [15] A. Cucchieri and D. Zwanziger, Phys. Rev. **D65** (2002) 014002

Figure Captions

Fig.1. The transverse gluon self-energy graph. The dashed line is the transverse gluon A_i , the dotted line represents the instantaneous Coulomb field A_0 and the continuous line is the E_i field conjugate to the transverse field A_i .

Fig.2. The transverse gluon self-energy graph. The dashed line is the transverse gluon field A_i .

Fig.3a. The $A_i A_0$ two-point function. The dotted line is the instantaneous Coulomb field A_0 , the dashed line represents the transverse field A_i and the solid line is the conjugate field E_i .

Fig.3b. The $A_i A_0$ two-point function. The graph is suppressed as an energy divergence.

Fig.4a. The time-time component of the gluon self-energy to order g^2 . The dotted lines represent the instantaneous Coulomb field A_0 . The continuous line is the E_i field conjugate to the transverse field A_i . The propagators inside the loop are the $E_i A_j$ transitions specific to the Coulomb gauge.

Fig.4b. Self-energy graph to order g^2 . The dotted line is the A_0 field, the dashed line is the transverse propagator and the solid line is the $E_i E_j$ propagator.

Fig.5. The transition between the transverse gluon field and its conjugate field E_i .

Fig.6. The transition between the Coulomb field A_0 and the conjugate field E_i .

Fig.7. The conjugate field self-energy.

Fig.8. The Slavnov-Taylor identity for self-energy graphs. The wavy lines stand for Yang-Mills particles and double lines for ghosts. The symbol on the left wavy line stands for the replacement of a polarization vector $e_\mu(k)$ by k_μ and k^2 need not be zero. The cross denotes the action of the tensor $(k_\mu k_\nu - k^2 \delta_{\mu\nu})$. The circle represents the set of all relevant Feynman graphs.

Fig.9a. Diagram with an open ghost line. The source v_n of the E_m field has the vertex $gf^{abc}E_n^b C^c v_n$. The ghost propagator is $\frac{1}{K^2}$ and it is represented with the double line.

Fig.9b. Diagram with an open ghost line. The source u_i^a of the transverse gluon field has the vertex $gf^{abc}\delta_{ij}$.























